Validation of Ionospheric Simulation with PALSAR

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November 2009
Overview

- Deriving the Faraday rotation through the ionosphere
  - Derivation from the physical parameters
  - Derivation using SAR data

- Describe the implementation of the homogeneous slab model
  - Model is valid for wide bandwidth and wide beamwidth

- Validated simulation of the homogenous slab ionosphere using PALSAR data

- Simulated imagery with different TEC levels

- Coherence at difference TEC levels
Modeling the Homogeneous Slab Ionosphere
Deriving the Faraday Rotation

- From the index of refraction of the ordinary and extraordinary waves \( (n_\pm) \), the Faraday Rotation can be derived in two ways:

  \[ n_\pm \rightarrow \Phi_\pm(\kappa, \omega) \rightarrow \Omega \]

  A direct derivation from physical parameters based on the difference in the phase paths for the two waves is possible.

  From the index of refraction one can derive the phase accumulated when travelling through the ionosphere for the two waves.

  The Faraday rotation can be computed directly from the SAR data.

  This accumulated phase can easily be applied to SAR data in order to simulate passing through the ionosphere.
Homogeneous Slab Ionosphere

• Assumptions
  – Spatially homogeneous
  – No temporal variations

• These spatial and temporal symmetries imply that the ionosphere’s effect on SAR phase history data can be simply captured in Fourier coordinates
  – Wavenumber $\kappa = \text{Fourier coordinate conjugate to along-track coordinate}$
  – Frequency $\omega = \text{Fourier coordinate conjugate to fast-time}$
Ionospheric Effect in \((\kappa, \omega)\)-Domain

- \(d(x,t) = \) phase history data \textit{without} ionosphere
- \(D(\kappa, \omega) = \) Fourier transform of \(d\)

- \(D_{\text{iono}}(\kappa, \omega) = \Phi(\kappa, \omega) \cdot D(\kappa, \omega)\)
  - Form of relationship follows from spatial and temporal symmetry assumed for ionosphere
  - Specifically, \(D_{\text{iono}}(\kappa, \omega)\) depends on \(D(\kappa', \omega')\) only for \(\kappa' = \kappa, \omega' = \omega\)

- \(d_{\text{iono}}(x,t) = \) inverse Fourier transform of \(D_{\text{iono}} = \) phase history data \textit{with} ionosphere

- Image formation applied to \(d_{\text{iono}}\) produces a SAR image with ionospheric effects
Slant-Range through Ionosphere

- TEC effectively measures “thickness” of slab ionosphere
- We need the slant-TEC over the synthetic aperture

\[ \text{slant}_\text{TEC} = \text{TEC} \sec \theta_{\text{inc}} \sec \phi_{\text{az}} \]
Relation between Azimuth Angle and \((\kappa, \omega)\)

\[\text{Doppler} = \kappa v = \frac{2\omega}{c} v \sin \phi_{az}\]

\[\Rightarrow \sec \phi_{az} = \frac{1}{\sqrt{1 - \left(\frac{c\kappa}{2\omega}\right)^2}}\]
Symbols

- $\omega$: fast-time (RF) frequency
- $\kappa$: along-track wavenumber
- $\theta_{inc}$: incidence angle
- $\phi_{az}$: azimuth angle (complement of Doppler cone angle)
- $\varphi$: angle between propagation and magnetic field
- $c$: speed of light
- $m$: mass of the electron ($9.109 \times 10^{-31}$ m)
- $e$: charge of the electron ($1.6 \times 10^{-19}$ C)
- $B$: magnetic field
- TEC: total electron count (vertically through ionosphere)
- slant_TEC: TEC for slanted path through ionosphere
- $\varepsilon_0, \mu_0$: electric and magnetic constant
Derivation of $n_\pm$ in the Ionosphere

- In a medium with dielectric constant, Maxwell’s equations in rationalized MKS units are:

$$\nabla \times H = \dot{D} = \varepsilon_0 \dot{E} + \dot{P} \quad \nabla \times E = -\mu_0 H \quad \nabla \cdot D = \nabla \cdot H = 0$$

- We consider a wave propagating like $\exp(2\pi i(\omega t-kx))$. The index of refraction can be derived [Xu, Wu, & Wu] from the above and the equations of motion. After setting some constants:

$$X = \frac{Ne^2}{(2\pi)^2 \varepsilon_0 m \omega^2} = \frac{\omega_p^2}{\omega^2} \quad Y_L = \frac{\mu_0 He}{2\pi m \omega} \cos \varphi = \frac{\omega_L}{\omega} \quad Y_T = \frac{\mu_0 He}{2\pi m \omega} \sin \varphi = \frac{\omega_T}{\omega}$$

- The index of refraction for ordinary (+) and extraordinary (-) waves is then:

$$n^2_\pm = 1 - \frac{X}{1 - \frac{Y_T^2}{2(1-X)} \pm \sqrt{\frac{Y_T^4}{4(1-X)^2} + Y_L^2}}$$
Index of Refraction in the Ionosphere (cont’d)

- For most angles ($\varphi < 86^\circ$), the quasi-longitudinal approximation applies:
  \[
  \frac{\sin^2 \varphi}{2 \cos \varphi} \ll \frac{\omega^2 - \omega_p^2}{\omega_c \omega}
  \]

- With, $\omega_p \sim 12$ MHz, $\omega_c \sim 1.4$ MHz, and $\omega \sim 230$ MHz – 40 GHz

- This means that the expression for the index of refraction can be simplified to:
  \[
  n^2 \pm 1 = 1 - \frac{X}{1 \pm Y_L}
  \]

- In the frequencies of interest, it can be expressed as:
  \[
  n^2 \pm 1 = -\frac{1}{2} X(1 \mp Y_L) = -\frac{Ne^2}{(2\pi)^2 \varepsilon_0 m \omega^2} \left(1 \mp \frac{\mu_0^2 eB \cos \varphi}{2\pi m \omega}\right)
  \]
Index of Refraction in the Ionosphere (cont’d)

- The +/- signs (ordinary/extraordinary) depend on the polarization and relationship to the magnetic field:
  - + for right circular propagating parallel to B
  - + for left circular propagating anti-parallel to B
  - - for left circular propagating parallel to B
  - - for right circular propagating anti-parallel to B
Faraday Rotation from $n_\pm$

- The Faraday rotation is equal to half the differential phase path length, multiplied by $2\pi/\lambda$ [Xu, Wu, & Wu]:

$$\Omega = \frac{\pi}{\lambda} \int (n_+ - n_-) ds = \frac{\pi}{\lambda} \int (XY_L) ds = \frac{\pi e^3}{(2\pi)^3 \lambda \varepsilon_0 m^2 \omega^3} \int NB \cos \varphi ds$$

$$= \left( \frac{e^3}{8\pi^2 \varepsilon_0 m^2 c} \right) \frac{1}{\omega^2} \int NB \cos \varphi \sec \theta_{inc} dh$$

$$\Rightarrow \Omega = \frac{K}{\omega^2} \int NB \cos \varphi \sec \theta_{inc} dh$$

with

$$K = \left( \frac{e^3}{8\pi^2 \varepsilon_0 m^2 c} \right) = 2.365 \times 10^4 \text{ A} \cdot \text{m}^2 \cdot \text{kg}^{-1}$$
Phase Delay from $n_\pm$

- The accumulated phase delay from the ionosphere can then be derived:

$$
\Phi_\pm(\omega, \kappa) = \int \frac{2\pi \omega}{c} (n_\pm - 1) ds = -\int \frac{\pi \omega X}{c} (1 + Y_L) ds
$$

$$
= -\int \frac{\pi \omega}{c} \left( \frac{N e^2}{(2\pi)^2 \varepsilon_0 m \omega^2} \right) \left( 1 + \frac{B e \cos \varphi}{2\pi m \omega} \right) ds
$$

$$
= -\frac{e^2 \sec \theta_{inc}}{4\pi \varepsilon_0 m \omega} \left( 1 + \frac{B e \cos \varphi}{2\pi m \omega} \right) \int N dh = -\frac{e^2 TEC \sec \phi_{az} \sec \theta_{inc}}{4\pi \varepsilon_0 m \omega} \left( 1 + \frac{B e \cos \varphi}{2\pi m \omega} \right)
$$

$$
\Rightarrow \Phi_\pm(\omega, \kappa) = -\frac{e^2 \sec \theta_{inc} TEC}{2\pi \varepsilon_0 m c^2} \left( 1 + \frac{B e \cos \varphi}{2\pi m \omega} \right) \left( \frac{1}{\sqrt{\left( \frac{2\omega}{c} \right)^2 - \kappa^2}} \right)
$$
Faraday Rotation from Phase Delay

- The Faraday rotation is equal to half the difference in the phase delay:

\[
\Omega = \frac{1}{2} (\Phi_+ - \Phi_-) = \int \frac{\pi \omega X Y_L}{c} ds
\]

\[
= \int \frac{\pi \omega}{c} \left( \frac{N e^2}{(2 \pi)^2 \epsilon_0 m \omega^2} \right) \left( \frac{B e \cos \varphi}{2 \pi m \omega} \right) ds
\]

\[
= \left( \frac{e^3}{8 \pi^2 \epsilon_0 m^2 c} \right) \frac{1}{\omega^2} \int NB \cos \varphi ds
\]

\[
\Rightarrow \Omega = \frac{K}{\omega^2} \int NB \cos \varphi \sec \theta_{inc} dh
\]

- This expression is the same as the one obtained by a direct derivation.
Calculating Faraday Rotation from SAR data

The Faraday Rotation is calculated from the SAR data using the following equations from Bickel and Bates [3]:

\[
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix} = \begin{bmatrix}
1 & i \\
i & 1
\end{bmatrix} \cdot \begin{bmatrix}
M_{hh} & M_{vh} \\
M_{hv} & M_{vv}
\end{bmatrix} \cdot \begin{bmatrix}
1 & i \\
i & 1
\end{bmatrix}
\]

\[\Omega = \frac{1}{4} \arg(Z_{12}Z_{21}^*)\]
Applying the Ionosphere Model to SAR data
Data Flow Diagram

Linear Polarized Calibrated Complex Imagery

→ Linear to Circular Polarization

→ Inverse SAR Processing

→ Apply Ionosphere Phase Shift in (κ, ω) domain

→ Uniform Slab Model

→ Forward SAR Processing

→ Circular to Linear Polarization

→ Linear Polarized Complex Imagery with Ionospheric Effects

→ Faraday Rotation Calculation
PALSAR example

- Data taken of Washington, DC on January 2007
- We can calculate the Faraday rotation with the physical parameters using:
  \[ \Omega = \frac{K}{\omega^2} \int NB \cos \phi \sec \theta_{inc} dh = \frac{K \cdot TEC \cdot B}{\omega^2} \cos \phi \sec \theta_{inc} \]
- **K**: \(2.365 \times 10^4\) A m\(^2\) kg\(^{-1}\)
- Center frequency: \(1.27 \times 10^9\) Hz
- Inclination angle: \(21.5^\circ\)
- \(\phi\): \(26.3^\circ\)
- TEC: \(12.5 \times 10^{16}\) electrons m\(^{-2}\) = 12.5 TECU
  - NOAA Geophysical Data Center Online calculator (www.ngdc.noaa.gov/IONO/USTEC/home.html)
- Magnetic field: \(4.3 \times 10^{-5}\) Tesla
  - NOAA Geophysical Data Center Online calculator (www.ngdc.noaa.gov/geomag/models.shtml)

\[ \Rightarrow \Omega = 4.44^\circ \]

Same as Meyer & Nicoll
And ASF Catalog
Original Imagery

Physically Calculated FR: 4.44°
Peak FR from data: 2.60°

Meyer & Nicoll, mean FR = 2.83°
Imagery corrected for Ionosphere Effects

Near range

Far range

HH

VV

Peak FR from data: 0.00°
Imagery Simulated with TEC = 7 TECU

Physically Calculated FR: 2.49°
Peak FR from data: 2.57°
Imagery Simulated with TEC = 12 TECU

Physically Calculated FR: 4.26°
Peak FR from data: 4.38°
Imagery Simulated with TEC = 30 TECU

Physically Calculated FR: 10.66°
Imagery Simulated with TEC = 60 TECU

Physically Calculated FR: 21.32°
Peak FR from data: 21.55°
Imagery Simulated with TEC = 100 TECU

Physically Calculated FR: 35.52°
Peak FR from data: 35.51°
Imagery Simulated with TEC = 126 TECU

Physically Calculated FR: $44.76^\circ$
Peak FR from data: $45.14^\circ$
Imagery Simulated with TEC = 150 TECU

Physically Calculated FR: 53.61°
Peak FR from data: 53.69°
-includes wraparound
Imagery Simulated with TEC = 200 TECU

Physically Calculated FR: 71.48°
Peak FR from data: 71.55°
-includes wraparound
Imagery Simulated with TEC = 253 TECU

Physically Calculated FR: 89.88°
Peak FR from data: 90.41°
-includes wraparound
Channel Mixing

\[
\begin{bmatrix}
M_{hh} & M_{vh} \\
M_{hv} & M_{vv}
\end{bmatrix}
= \begin{bmatrix}
\cos \Omega & \sin \Omega \\
-\sin \Omega & \cos \Omega
\end{bmatrix}
\begin{bmatrix}
S_{hh} & S_{vh} \\
S_{hv} & S_{vv}
\end{bmatrix}
\begin{bmatrix}
\cos \Omega & \sin \Omega \\
-\sin \Omega & \cos \Omega
\end{bmatrix}
\]

\[
M_{hh} = \cos^2 \Omega S_{hh} + \cos \Omega \sin \Omega S_{hv} - \cos \Omega \sin \Omega S_{vh} - \sin^2 \Omega S_{vv}
\]

\[
M_{vh} = \cos \Omega \sin \Omega S_{hh} + \sin^2 \Omega S_{hv} + \cos^2 \Omega S_{vh} + \cos \Omega \sin \Omega S_{vv}
\]

\[
M_{hv} = -\cos \Omega \sin \Omega S_{hh} + \cos^2 \Omega S_{hv} + \sin^2 \Omega S_{vh} - \cos \Omega \sin \Omega S_{vv}
\]

\[
M_{vv} = -\sin^2 \Omega S_{hh} + \cos \Omega \sin \Omega S_{hv} - \cos \Omega \sin \Omega S_{vh} + \cos^2 \Omega S_{vv}
\]

- This implies that at FR of 45°, all channels are equally mixed, and FR of 90°, the HH and VV channels have switched.

- Ionospheric blurring is not expected for the narrow bandwidth of PALSAR, but this changing channel mixing through different TEC levels can produce the same effect.
Channel Mixing (cont’d)

No Faraday Rotation

Simulated Faraday Rotation 90°

Features only in HH will switch channels after 90° Faraday rotation
Verifying the Range Shift

- We can calculate the range shift per TECU from the simulated images.

![TEC 0](image1) ![TEC 253](image2)

- The above chart was compiled by picking out an easily identifiable point in each image (the tip of the island) and finding its pixel location at each level of simulated TEC. The pixel shift is converted to a distance using the pixel width of 9.4 meters.

- The slope of the fit is $\Delta r = 0.27$ meters per TECU.
Verifying the Range Shift (cont’d)

- We can also calculate the expected range shift per TECU analytically.
- For PALSAR, the range shift per simulated TEC \((\Delta r)\) is:

\[
\Delta r = c \tau \pm = \frac{C}{2\pi} \frac{\delta \Phi}{\delta \omega} \bigg|_{\omega = \omega_0} = \frac{e^2 \sec \theta_{inc} \text{T}ECU}{8\pi^2 \varepsilon_0 m \omega_0^3} \left( \omega_0 + \frac{Be \cos \varphi}{\pi m} \right) = \begin{cases} 0.268 \frac{m}{\text{T}ECU} & \text{if } + \\ 0.269 \frac{m}{\text{T}ECU} & \text{if } - \end{cases}
\]

- The overall expected shift in the HH imagery should be the average of + and - (transmit and receive):

\[
\Delta r = 0.27 \frac{m}{\text{T}ECU}
\]

- This compares favorable with the value of 0.27 from the images.

- Since geometric accuracy of PALSAR is \(\sim 10\text{m}\), TEC > 40 TECU for range shift to be observed by PALSAR.
Coherence From Simulated Data
Coherence Maps, 0 TEC compared to 7 TEC

HH

VV

$\mu = 0.967$

$\sigma = 0.011$

$\mu = 0.968$

$\sigma = 0.010$

5km x 5km
Coherence Maps, 0 TEC compared to 12 TEC

HH

VV
Coherence Maps, 0 TEC compared to 60 TEC

HH

VV

near range

far range

Normalized Count

Coherence

\[ \mu = 0.900 \]

\[ \sigma = 0.034 \]

Normalized Count

Coherence

\[ \mu = 0.897 \]

\[ \sigma = 0.036 \]

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Coherence Maps, 0 TEC compared to 126 TEC (FR = 45°)
Coherence Maps, 0 TEC compared to 150 TEC
Coherence Maps, 0 TEC compared to 253 TEC (FR = 90°)

Area of high coherence over water
Coherence Map, Original Imagery HH to VV

\[ \mu = 0.388 \]
\[ \sigma = 0.188 \]
Conclusions

• Demonstrated that computation of Faraday Rotation from physical parameters and computation from SAR data are equivalent

• Successfully validated our slab ionospheric model using PALSAR data
References

